QP CODE: 25800347

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, JANUARY 2025

First Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

Complementary - IPH1CM05 - MATHEMATICS - I DIFFERENTIAL CALCULUS, MATRICES AND TRIGONOMETRY

2021 Admission Onwards

A92A66CF

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the slope of the curve $x^2 + y^2 = 25$ through the point $(3, -4)$.
2. State L'Hôpital's rule.
3. Find the domain and range of the function $f(x, y) = \frac{1}{\sqrt{16-x^2-y^2}}$.
4. Find f_x, f_y and f_z if $f(x, y, z) = e^{-(x^2+y^2+z^2)}$.
5. Draw the tree diagram and write the chain rule formula for the function $z = f(x, y), x = g(t), y = h(t)$.
6. Differentiate beween equal and equivalent matrices.
7. Find the eigen values of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
8. If x and y are real or complex numbers, prove that $\sin 2x = 2sinx\ cosx$.
9. Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$.
10. Write the expansion of e^x and $cos\ x$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the absolute maximum and absolute minimum values of $f(x) = 4 - x^2$ on $-3 \leq x \leq 1$.
12. a.Find the critical points of $f(x) = (x^2 - 3)e^x$.
    b.Identify the intervals on which f is increasing and decreasing.
    c. Find the local and absolute extreme values of $f(x)$.
13. Verify whether the function $w = 5\cos(3x + 3ct) + e^{x+ct}$ satisfies the wave equation.

14. Use chain rule to evaluate $\frac{dw}{dt}$ at $t = 1$ if $w = z - \sin(xy),\ x = t,\ y = \ln t,\ z = e^{t-1}$.

15. If A and B are n-square skew-symmetric matrices, then AB is symmetric if and only if A and B commute.

16. Apply L'Hôpital's rule to find $\lim_{x\to\infty} \frac{5x^2-2x}{7x^3+3}$.

17. Separate into real and imaginary parts the quantity $\sin^{-1}(\cos\theta + i\sin\theta)$, where θ is real.

18. Show that the sum of the infinite series $\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1.3}{2.4}\cos 3\theta + \frac{1.3.5}{2.4.6}\cos 4\theta + \ldots$ is $\cos(\frac{\pi}{4} + \frac{3\theta}{4})(2\sin\frac{\theta}{2})^{-\frac{1}{2}}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ if $x^3 + z^2 + ye^{xz} + z\cos y = 0$..
b. Express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ in terms of u and v if $w = \ln(x^2 + y^2 + z^2),\ x = ue^v \sin u,\ y = ue^v \cos u,\ z = ue^v$. Also evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(-2, 0)$.

20. Investigate for what values of λ ,μ the simultaneous equations
$$x + 2y + z = 8$$
$$2x + y + 3z = 13$$
$$3x + 4y - \lambda z = \mu$$
have a) no solution b) unique solution c) infinitely many solutions.

21. Solve the following
$$4x + y + 2z = 0$$
$$-3x + 2y + 4z = 0$$
$$8x - y - 2z = 0$$

22. a. Prove that $64\sin^3\theta\cos^4\theta = 3\sin\theta + 3\sin 3\theta - \sin 5\theta - \sin 7\theta$.
b. Expand $\sin^7\theta$ in a series of sines of multiples of θ.

(2×5=10 weightage)